2316 **175**

Class – B.A./B.Sc. I Sem II

Subject – Mathematics

Paper- P-II (Calculus-II)

Time Allowed : 3 Hours **Maximum Marks : 50**

Note:- Attempt any five questions selecting at least two from each section.

Section-A

1. (a) Show that for the function f defined by $f(x,y) = \dfrac{x^2y^2}{x^2y^2+(x-y)^2}$ the two repeated limits exist and are equal but the simultaneous limit doesn't exist.

(b) Let $f : R^2 \to R$ be defined by $f(x,y) = \begin{cases} 1, \text{ if } x \text{ is irrational} \\ 0, \text{ if } x \text{ is rational} \end{cases}$ Show that for any point (x, y), $\lim_{(x,y)\to(x_0,y_0)} f(x,y)$ doesn't exist. 5,5

2. (a) Discuss the continuity of the function f (x,y) defined by $f(x,y) = \begin{cases} xy\left(\dfrac{x^2-y^2}{x^2+y^2}\right), & (x,y)\neq(0,0) \\ 0, & (x,y)=(0,0) \end{cases}$ at origin.

(b) Use Taylor's Theorem for functions of two variables to expand $x^2y + 3y - 2$ in the powers of $x - 1$ and $y+2$. 5,5

3. (a) Let $f(x,y) = \dfrac{xy(x-y)}{x+y}$, where $(x, y) \neq (0, 0)$ & $f(0, 0) = 0$ show that $f_{xy}(0,0) = f_{yx}(0,0)$

(b) If $u = x f\left(\dfrac{y}{x}\right) + y\,\phi\left(\dfrac{y}{x}\right)$, prove that

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = 0$$ 5,5

4. (a) If $u^3 = xyz$, $\dfrac{1}{v} = \dfrac{1}{x} + \dfrac{1}{y} + \dfrac{1}{z}$ and

$w = x^2 + y^2 + z^2$; prove that $\dfrac{\partial(u,v,w)}{\partial(x,y,z)} =$

$$-\frac{v(y-z)(z-x)(x-y)(x+y+z)}{3u^2w(yz+zx+xy)}$$

(b) Show that the functions $u = x + y - z$, $v = x - y + z$ and $w = x^2 + y^2 + z^2 - 2yz$ are not independent of one another. Also find the relation between them. 5,5

Section-B

5. (a) Evaluate $\iint x^{1/2} y^{1/2} (1-x-y)^3\, dx\, dy$, over the region A bounded by the triangle whose vertices are (0,0), (1,0) & (0,1)

(b) Change the order of integration of

$$\int_{-a}^{a} \int_{\frac{1}{2}\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}} f(x,y)\, dy\, dx$$

Hence evaluate it when f (x,y) = 1 4,6

6. (a) Find the volume common to cylinders $x^2 + y^2 = a^2$ and $x^2 + z^2 = a^2$

(b) Evaluate $\iint_E \text{Sin}\left(\frac{x-y}{x+y}\right) dx\, dy$, where E is the region bounded the co-ordinates & x + y = 1 is the Ist quadrant.

5,5

7. (a) Prove that $\iiint_V \sqrt{1-\frac{x^2}{a^2}-\frac{y^2}{b^2}-\frac{z^2}{c^2}}\; dx\, dy\, dz = \frac{\pi^2 abc}{4}$,

Where $V = \left\{ (x, y, z) \Big/ \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} \le 1 \right\}$

(b) Show that $\iiint \left(x^2 + y^2 + z^2 \right) dx\, dy\, dz = \frac{4\pi}{5}$, over the region $x^2 + y^2 + z^2 \le 1$. 5,5

8. (a) Find the area enclosed by the parabolas $y^2 = 4ax$ & $x^2 = 4ay$, $a > 0$.

(b) Show that the entire volume of the solid

$$\left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{b}\right)^{2/3} + \left(\frac{z}{c}\right)^{2/3} = 1 \text{ is } \frac{4\pi abc}{35}$$ 4,6

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